

# On neutral scalar radiation by a massive orbiting star in extremal Kerr-Newman black hole

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## Abstract

In this short note we extend the work of 1401.3746 about gravity waves by a massive orbiting star in an extremal Kerr black hole to an extremal Kerr-Newman black hole for scalar radiation, and still find that it has a CFT interpretation from Kerr-Newman/CFT. In addition, we investigate on electromagnetic radiation with Kerr/CFT, which a detailed analysis isn't given by 1401.3746.

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# 1 Introduction

Detecting gravitational waves is an important work, recently BICEP2[1] found primordial gravitational waves which provide a more firmer evidence on inflation. Black hole radiation is also the source of gravitational waves, recently [2][3] make use of conformal symmetry studying gravitational radiation produced by a massive orbiting star very near the horizon of an extreme Kerr black hole, they find analytic results and provide a new evidence on Kerr/CFT conjecture[4], which is that bulk quantum gravity on the NHEK(Near-Horizon Extreme Kerr) geometry of Kerr is dual to a two-dimensional boundary CFT. The orbiting star sits at  $r_0$  along the radial direction of Kerr black hole, where the boundary at  $r \rightarrow \infty$ . The orbiting star is a perturbative source and breaks the scaling symmetry of the bulk theory, however if we assume that there is a renormalization group flow between UV CFT and IR CFT on the boundary, then according to holographic renormalization group flow[5][6][7], so the bulk theory of  $r < r_0$  corresponds to IR CFT. This is what [2] did, then the gravitational radiation follows from the application of Fermis golden rule to the IR CFT, they get a nontrivial agreement between bulk and boundary calculations.

One simple extension of Kerr/CFT is Kerr-Newman/CFT [8], in [9] the authors calculate near-superradiant scattering of charged scalars and fermions in a near-extreme Kerr-Newman black hole and find that Kerr-Newman/CFT is also hold in this case. Now we want to generalize the consideration in[2] simply to Kerr-Newman case and make a small check on Kerr-Newman/CFT conjecture. For simplicity, we also consider a massless neutral scalar field radiation, then we find that the bulk gravitational radiation is also agreement with the CFT result.

For electromagnetic and gravitational perturbations in Kerr black hole, Teukolsky [17][18][19][20] find using Newman-Penrose formalism [21], the equation of perturbation is decoupled into angular part and radial part. For the NHEK geometry, the similar analysis is done in [22][12], we will use their result to analyze the electromagnetic perturbation along with 1401.3746.

To make the note more readable, in section 2 we briefly review Kerr-Newman black hole, NHEKN and NHEKN/CFT. In section 3 we calculate the massless neutral scalar radiation both from the bulk gravity theory and the boundary CFT. In section 4 we

give the calculation of electromagnetic perturbation with Kerr/CFT. The last section is some discussion.

## 2 Review Kerr-Newman black hole, NHEKN and NHEKN/CFT

The metric of the Kerr-Newman black hole with mass  $M$ , angular momentum  $J = aM$ , and electric charge  $Q$  in the Boyer-Lindquist coordinates is ( $G = \hbar = c = 1$ )[9]:

$$ds^2 = -\frac{\Delta}{\hat{\rho}^2} \left( d\hat{t} - a \sin^2 \theta d\hat{\phi} \right)^2 + \frac{\sin^2 \theta}{\hat{\rho}^2} \left( (\hat{r}^2 + a^2) d\hat{\phi} - a d\hat{t} \right)^2 + \frac{\hat{\rho}^2}{\Delta} d\hat{r}^2 + \hat{\rho}^2 d\theta^2 \quad (1)$$

where

$$\Delta = \hat{r}^2 + a^2 - 2M\hat{r} + Q^2, \quad \hat{\rho}^2 = \hat{r}^2 + a^2 \cos^2 \theta \quad (2)$$

And the gauge field and field strength are

$$\begin{aligned} A &= -\frac{Q\hat{r}}{\hat{\rho}^2} (d\hat{t} - a \sin^2 \theta d\hat{\phi}) \\ F &= -\frac{Q(\hat{r}^2 - a^2 \cos^2 \theta)}{\hat{\rho}^4} (d\hat{t} - a \sin^2 \theta d\hat{\phi}) \wedge d\hat{r} \\ &\quad - \frac{2Q\hat{r}a \cos \theta}{\hat{\rho}^4} \sin \theta d\theta \wedge (a d\hat{t} - (\hat{r}^2 + a^2) d\hat{\phi}) \end{aligned} \quad (3)$$

The horizon  $r_{\pm}$  are

$$r_{\pm} = M \pm \sqrt{M^2 - a^2 - Q^2} \quad (4)$$

The entropy, Hawking temperature, angular velocity of the horizon, and electric potential are

$$\begin{aligned} S &= \frac{\text{Area}}{4} = \pi(r_+^2 + a^2) \\ T_H &= \frac{r_+ - r_-}{4\pi(r_+^2 + a^2)} \\ \Omega_H &= \frac{a}{r_+^2 + a^2} \\ \Phi &= \frac{Qr_+}{r_+^2 + a^2} \end{aligned} \quad (5)$$

We also define the dimensionless Hawking temperature

$$\tau_H \equiv \frac{r_+ - r_-}{r_+} \quad (6)$$

Now we consider near-horizon extremal Kerr-Newman geometry. The extremal black hole has  $r_+ = r_- = M$ , then we take near horizon limit by defining[10][8]

$$\begin{aligned}\hat{r} &= r_+ + \lambda r_0 r , \\ \hat{t} &= t r_0 / \lambda , \\ \hat{\phi} &= \phi + \Omega_H \frac{t r_0}{\lambda}\end{aligned}\tag{7}$$

with  $r_0^2 = r_+ + a^2$ . Taking  $\lambda \rightarrow 0$ , the near horizon geometry is

$$ds^2 = \Gamma(\theta) \left[ -r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 \right] + \gamma(\theta) (d\phi + b r dt)^2\tag{8}$$

where

$$\begin{aligned}\Gamma(\theta) &= r_+^2 + a^2 \cos^2 \theta \\ \gamma(\theta) &= \frac{(r_+^2 + a^2)^2 \sin^2 \theta}{r_+^2 + a^2 \cos^2 \theta} \\ b &= \frac{2ar_+}{r_+^2 + a^2}\end{aligned}\tag{9}$$

NHEKN has an enhanced isometry group  $U(1)_L \times SL(2, R)_R$ , generated by the Killing vectors

$$\begin{aligned}Q_0 &= \partial_\phi , \\ H_1 &= \partial_t , \\ H_2 &= t\partial_t - r\partial_r \\ H_3 &= \left( \frac{1}{2r^2} + \frac{t^2}{2} \right) \partial_t - tr\partial_r - \frac{b}{r} \partial_\phi\end{aligned}\tag{10}$$

The first law of thermodynamics for Kerr-Newman is

$$T_H dS = dM - \Omega_H dJ - \Phi dQ\tag{11}$$

At extremality,  $T_H = 0$ , the above formula reduces to  $dM = \Omega_H dJ + \Phi dQ$ , the first law becomes

$$dS = \frac{1}{T_L} (dJ - \mu_L dQ)\tag{12}$$

where[2]

$$\begin{aligned}T_L &= \frac{r_+^2 + a^2}{4\pi J} \\ \mu_L &= -\frac{Q^3}{2J}\end{aligned}\tag{13}$$

According to the Kerr/CFT correspondence,  $T_L$  is the left moving temperature of the dual 2D CFT.

### 3 Scalar radiation from a star orbiting near the horizon

In this section we calculate a massless neutral scalar radiation caused by a massive orbiting star in the NHEKN geometry from the bulk gravity, and compare with the result of the boundary CFT. We find that both calculations agree with each other well, so this supports the Kerr-Newman/CFT conjecture.

#### 3.1 Gravity analysis in NHEKN

There is a star orbiting at radius  $r_0$  in the NHEKN geometry (8). We parameterize the corresponding geodesic  $x_*^\mu(t)$  with the NHEKN time  $t$ :

$$\begin{aligned} x_*^t(t) &= t, \\ x_*^\phi(t) &= \phi_0 - \omega_s r_0 t, \\ x_*^r(t) &= r_0, \\ x_*^\theta(t) &= \frac{\pi}{2}. \end{aligned} \tag{14}$$

where the velocity  $\omega_s$  of the orbiting star

$$\omega_s = \frac{(4a^2 - M^2)M}{2a(M^2 + a^2)} \tag{15}$$

The orbiting star is moving along its geodesic in the NHEKN geometry, its energy is zero

$$E = -g_{t\mu} \partial_\tau x_*^\mu = 0 \tag{16}$$

where  $\tau$  is the proper time. The angular momentum (per unit mass) is

$$L = g_{\phi\mu} \partial_\tau x_*^\mu = \frac{M^2 + a^2}{\sqrt{4a^2 - M^2}} \tag{17}$$

We couple the star to a massless neutral scalar with the interaction

$$S_I = 4\pi\lambda \int d\tau \Psi(x_*(\tau)) \tag{18}$$

where  $\lambda$  is a coupling constant. Then the scalar wave equation in the presence of the star is:

$$\square \Psi = 4\pi\mathcal{T} \tag{19}$$

where (setting  $\phi_0 = 0$ )

$$\begin{aligned}
\mathcal{T} &= -\lambda(-g)^{-\frac{1}{2}} \frac{d\tau}{dt} \delta(r - r_0) \delta(\theta - \frac{\pi}{2}) \delta(\phi + \omega_s r_0 t) \\
&= -\lambda \frac{M\sqrt{4a^2 - M^2}}{2a} r_0 \Gamma^3(\theta) \gamma(\theta) \Big|_{\theta=\frac{\pi}{2}} \delta(r - r_0) \delta(\theta - \frac{\pi}{2}) \delta(\phi + \omega_s r_0 t) \\
&= -\lambda \frac{\sqrt{4a^2 - M^2}}{2aM(M^2 + a^2)} r_0 \delta(r - r_0) \delta(\theta - \frac{\pi}{2}) \delta(\phi + \omega_s r_0 t)
\end{aligned} \tag{20}$$

where  $g$  is the determinant of the metric. The orbiting star preserve the Killing symmetry:

$$\chi = \partial_t - \omega_s r_0 \partial_\phi \tag{21}$$

Then a  $\chi$ -invariant solution to the wave equation is

$$\Psi = \sum_{\ell, m} e^{im(\phi + \omega_s r_0 t)} S_\ell(\theta) R_{\ell m}(r), \tag{22}$$

$$\mathcal{T} = \frac{1}{4\pi(M^2 + a^2 \cos^2 \theta)} \sum_{\ell, m} e^{im(\phi + \omega_s r_0 t)} S_\ell(\theta) T_{\ell m}(r) \tag{23}$$

where  $S_\ell$  are the spheroidal harmonics obeying

$$\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta S_\ell) + \left( K_\ell - \frac{m^2}{\sin^2 \theta} - \frac{a^4 m^2}{(r_+^2 + a^2)^2} \sin^2 \theta \right) S_\ell = 0 \tag{24}$$

with  $K_\ell$  is a separation constant,  $\ell \geq 0, -\ell \leq m \leq \ell$ . Note that  $S_\ell$  and  $K_\ell$  depend on both  $m$  and  $\ell$ . The spheroidal harmonics satisfy the normalization condition:

$$\int_0^\pi \sin \theta d\theta S_\ell(\theta) S_{\ell'}(\theta) = \delta_{\ell\ell'} \tag{25}$$

The coefficients of  $\mathcal{T}$  are

$$T_{\ell m}(r) = -\lambda \frac{M\sqrt{4a^2 - M^2}}{a(M^2 + a^2)} r_0 S_\ell(\pi/2) \delta(r - r_0) \tag{26}$$

The separated radial equation is

$$\partial_r (r^2 \partial_r R_{\ell m}) + \left( \frac{\omega^2}{r^2} + c_2 \frac{2\omega m}{r} + \frac{(2ar_+)^2}{(r_+^2 + a^2)^2} m^2 + \frac{2a^2}{r_+^2 + a^2} m^2 - K_\ell \right) R_{\ell m} = T_{\ell m} \tag{27}$$

where  $c_2 = \frac{2ar_+}{r_+^2 + a^2}$  and  $\omega = -m\omega_s r_0$

There are two linear independent solutions to the homogeneous radial equation ( $T_{\ell m} = 0$ ), they are the Whittaker functions[12]:

$$M_{imc_2, h-\frac{1}{2}}(-2i\omega/r), \quad W_{imc_2, h-\frac{1}{2}}(-2i\omega/r) \tag{28}$$

where

$$h \equiv \frac{1}{2} + \sqrt{1/4 + K_\ell - \frac{2a^2(3r_+^2 + a^2)}{(r_+^2 + a^2)^2} m^2} \quad (29)$$

We also consider the case  $h$  is real and  $h < 1$  as in [2].

We then find the solution of (27) that is purely ingoing at the horizon and obeys Dirichlet boundary condition at  $r = \infty$ , see the similar process in [2]

$$R_{\ell m}(r) = \frac{1}{W} \left[ X \Theta(r_0 - r) W_{imc_2, h - \frac{1}{2}}(-2i\omega/r) + Z \Theta(r - r_0) M_{imc_2, h - \frac{1}{2}}(-2i\omega/r) \right] \quad (30)$$

The Wronskian  $W$  of the two linear independent solution (28),

$$W = 2i\omega \frac{\Gamma(2h)}{\Gamma(h - imc_2)} \quad (31)$$

and

$$\begin{aligned} X &= -\lambda \frac{M\sqrt{4a^2 - M^2}}{a(M^2 + a^2)} r_0 S_\ell(\pi/2) M_{imc_2, h - \frac{1}{2}}(2i\omega_s m) \\ Z &= -\lambda \frac{M\sqrt{4a^2 - M^2}}{a(M^2 + a^2)} r_0 S_\ell(\pi/2) W_{imc_2, h - \frac{1}{2}}(2i\omega_s m) \end{aligned} \quad (32)$$

So it is easy shown that the Klein-Gordon particle number flux

$$\mathcal{F} = - \int \sqrt{-g} J^r d\theta d\phi, \quad J^\mu = \frac{i}{8\pi} (\Psi^* \nabla^\mu \Psi - \Psi \nabla^\mu \Psi^*), \quad \nabla_\mu J^\mu = \text{Im}[\Psi \mathcal{T}] \quad (33)$$

is vanishing at the infinity for real  $h$ , while at the horizon we get

$$\begin{aligned} \mathcal{F}_{\ell m} &= -(r_+^2 + a^2) \frac{\omega}{2} \left| \frac{X_{\ell m}}{W_{\ell m}} \right|^2 e^{-\pi m c_2} \\ &= \frac{\lambda^2 M^2 (4a^2 - M^2) r_0}{8m\omega_s a^2 (M^2 + a^2)} S_\ell^2(\pi/2) \left| M_{imc_2, h - \frac{1}{2}}(2i\omega_s m) \right|^2 \\ &\quad \left| \frac{\Gamma(h - imc_2)}{\Gamma(2h)} \right|^2 e^{-\pi m c_2} \end{aligned} \quad (34)$$

The above formula gives the particle number flux across the future horizon.

### 3.2 CFT analysis

In this section we calculate the particle number flux from the boundary CFT in the spirit of [9], follows from AdS/CFT[13][14], the massless neutral scalar field in the bulk deforms the boundary CFT to

$$S = S_{CFT} + \sum_\ell \int dt^+ dt^- J_\ell(t^+, t^-) \mathcal{O}_\ell(t^+, t^-). \quad (35)$$

It was shown that [2]

$$J_\ell(\phi, t) = \sum_m \frac{X}{W} C e^{im(\phi + \omega_s r_0 t^-)} \quad (36)$$

where we identify that

$$t^+ = \phi \quad t^- = t \quad C = (-2i\omega)^{1-h} \frac{\Gamma(2h-1)}{\Gamma(h-imc_2)} \quad (37)$$

And the operator  $\mathcal{O}$  dual to the scalar field has conformal weight [11]:

$$h_L = h_R = h \quad (38)$$

where  $h$  is equal to (29). Then by using Fermis golden rule, the transition rate [15][16]

$$\mathcal{R} = 2\pi \sum_{\ell, m} |J_{\ell m}|^2 \int dt^+ dt^- e^{-imt^+ - im3r_0 t^-/4} G(t^+, t^-). \quad (39)$$

where  $G(t^+, t^-) = \langle \mathcal{O}^\dagger(t^+, t^-) \mathcal{O}(0, 0) \rangle_{T_L}$  is the two point function of the dual two-dimensional conformal field theory, its specific form is[9]

$$G = C_{\mathcal{O}}^2 (-1)^{h_L + h_R} \left( \frac{\pi T_L}{\sinh(\pi T_L t^+)} \right)^{2h_L} \left( \frac{\pi T_R}{\sinh(\pi T_R t^-)} \right)^{2h_R} e^{iq_L \mu_L t^+ + iq_R \mu_R t^-} \quad (40)$$

where  $C_{\mathcal{O}}$  is a normalization constant.

So from Kerr-Newman/CFT, we have  $q_L = e$ ,  $\mu_L = -\frac{Q^3}{2J}$ ,  $T_L = \frac{M^2 + a^2}{4\pi M a}$ , there is

$$\begin{aligned} R_{\ell m} &= C_{\mathcal{O}}^2 2\pi \frac{|X_{\ell m}|^2}{(2h-1)^2} (4\omega^2)^{-h} \frac{(2\pi T_L)^{2h-1}}{\Gamma(2h)^2} e^{-\frac{m}{2T_L}} |\Gamma(h + i\frac{m}{2\pi T_L})|^2 \\ &\quad (2\pi T_R)^{2h-1} e^{\frac{m\omega_s r_0 - q_R \mu_R}{2T_R}} |\Gamma(h + i\frac{m\omega_s r_0 - q_R \mu_R}{2\pi T_R})|^2 \\ &= C_{\mathcal{O}}^2 \frac{2\pi}{(2h-1)^2} \frac{\lambda^2 M^2 (4a^2 - M^2)}{a^2 (M^2 + a^2)^2} r_0^2 S_\ell^2(\pi/2) |M_{imc_2, h-\frac{1}{2}}(2i\omega_s m)|^2 \frac{(2\pi T_L)^{2h-1}}{\Gamma(2h)^2 (4\omega_s^2 m^2 r_0^2)^h} e^{-\frac{m}{2T_L}} \\ &\quad |\Gamma(h + i\frac{m}{2\pi T_L})|^2 (2\pi T_R)^{2h-1} e^{\frac{m\omega_s r_0 - q_R \mu_R}{2T_R}} |\Gamma(h + i\frac{m\omega_s r_0 - q_R \mu_R}{2\pi T_R})|^2 \\ &= C_{\mathcal{O}}^2 \frac{2\pi}{(2h-1)^2} \frac{\lambda^2 M^2 (4a^2 - M^2)}{a^2 (M^2 + a^2)^2} r_0^2 S_\ell^2(\pi/2) |M_{imc_2, h-\frac{1}{2}}(2i\omega_s m)|^2 \frac{(2\pi T_L)^{2h-1}}{\Gamma(2h)^2 (4\omega_s^2 m^2 r_0^2)^h} e^{-\pi m c_2} \\ &\quad |\Gamma(h + imc_2)|^2 (2\pi T_R)^{2h-1} e^{\frac{m\omega_s r_0 - q_R \mu_R}{2T_R}} |\Gamma(h + i\frac{m\omega_s r_0 - q_R \mu_R}{2\pi T_R})|^2 \end{aligned} \quad (41)$$

where we have set  $q_L = 0$ , because we consider a neutral scalar.

And in the limit  $T_R \rightarrow 0$ , we have

$$\begin{aligned} R_{\ell m} &= C_{\mathcal{O}}^2 \frac{2\pi}{(2h-1)^2} \frac{\lambda^2 M^2 (4a^2 - M^2)}{a^2 (M^2 + a^2)^2} r_0^2 S_\ell^2(\pi/2) |M_{imc_2, h-\frac{1}{2}}(2i\omega_s m)|^2 \frac{(2\pi T_L)^{2h-1}}{\Gamma(2h)^2 (4\omega_s^2 m^2 r_0^2)^h} e^{-\pi m c_2} \\ &\quad |\Gamma(h + imc_2)|^2 p_R^{2h-1} 2\pi \end{aligned} \quad (42)$$



where we have define  $p_R = m\omega_s r_0 - q_R \mu_R$ . If we assume  $q_R \mu_R \ll 1$  as [2], the above formula becomes to

$$\begin{aligned}
R_{\ell m} &= C_{\mathcal{O}}^2 \frac{(2\pi)^2}{(2h-1)^2} \frac{\lambda^2 M^2 (4a^2 - M^2)}{a^2 (M^2 + a^2)^2} r_0^2 S_\ell^2(\pi/2) |M_{imc_2, h-\frac{1}{2}}(2i\omega_s m)|^2 \frac{1}{2^{2h} \omega_s m r_0} \left(\frac{M^2 + a^2}{2Ma}\right)^{2h-1} e^{-\pi m c_2} \\
&\quad \frac{|\Gamma(h + imc_2)|^2}{\Gamma(2h)^2} \\
&= C_{\mathcal{O}}^2 \frac{(2\pi)^2}{(2h-1)^2} \frac{1}{2^{2h}} \left(\frac{M^2 + a^2}{2Ma}\right)^{2h-1} \frac{\lambda^2 M^2 (4a^2 - M^2) r_0}{a^2 (M^2 + a^2)^2 \omega_s m} e^{-\pi m c_2} S_\ell^2(\pi/2) |M_{imc_2, h-\frac{1}{2}}(2i\omega_s m)|^2 \\
&\quad \frac{|\Gamma(h + imc_2)|^2}{\Gamma(2h)^2}
\end{aligned} \tag{43}$$

### 3.3 Comparison

If we take

$$C_{\mathcal{O}}^2 = \frac{2^{2h} (2h-1)^2}{(2\pi)^2} \left(\frac{2Ma}{M^2 + a^2}\right)^{2h-1} \frac{1}{8} (M^2 + a^2) \tag{44}$$

the particle flux number in both calculations are exactly equal, i.e.  $\mathcal{F}_{\ell m} = R_{\ell m}$ . And when the charge  $Q$  of Kerr-Newman black hole is vanishing, we can also recover the result in [2].

## 4 Electromagnetic radiation in Kerr/CFT

Using null tetrad  $(l, n, m, \bar{m})$ , the scalar of electromagnetic field

$$\phi_2 = F_{\mu\nu} \bar{m}^\mu n^\nu \tag{45}$$

obeys the following perturbation equation

$$[(\Delta + \gamma - \bar{\gamma} + 2\mu + \bar{\mu})(D + 2\epsilon - \rho) - (\bar{\delta} + \alpha + \bar{\beta} + 2\pi - \bar{\tau})(\delta - \tau + 2\beta)] \delta\phi_2 = -2\pi J_2 \tag{46}$$

where

$$J_2 = (\Delta + \gamma - \bar{\gamma} + 2\mu + \bar{\mu}) J_{\bar{m}} - (\bar{\delta} + \alpha + \bar{\beta} + 2\pi - \bar{\tau}) J_n \tag{47}$$

The electromagnetic current of the orbiting star with charge  $q$  is

$$J^\nu = q \int d\tau (-g)^{-1/2} \frac{dx^\nu}{d\tau} \delta^{(4)}(x^\alpha - x_*^\alpha(\tau)) \tag{48}$$

For NHEK, the Kinnersley tetrad and nonzero spin coefficients can be seen in [2].

The nonzero component of the current along the geodesic

$$x_*^t(t) = t,$$

$$\begin{aligned}
x_*^\phi(t) &= -\frac{3}{4}r_0 t, \\
x_*^r(t) &= r_0, \\
x_*^\theta(t) &= \frac{\pi}{2}.
\end{aligned} \tag{49}$$

is

$$J_\phi = \frac{r_0 q}{2M^2} \delta^{(4)}(x^\alpha - x_*^\alpha(\tau)) \tag{50}$$

From (47) we have

$$\begin{aligned}
J_2 &= \frac{ir_0^2 q}{8\sqrt{2}M^5} \left[ 4\delta(r-r_0)\delta(\theta-\pi/2)\delta(\phi+3r_0 t/4) \right. \\
&\quad + r_0 \delta'(r-r_0)\delta(\theta-\pi/2)\delta(\phi+3r_0 t/4) \\
&\quad - \frac{3}{4}\delta(r-r_0)\delta(\theta-\pi/2)\delta'(\phi+3r_0 t/4) \\
&\quad \left. - 2i\delta(r-r_0)\delta'(\theta-\pi/2)\delta(\phi+3r_0 t/4) \right] \tag{51}
\end{aligned}$$

The equation (46) is separable for the variable

$$\Psi^{(-1)} \equiv \eta^{-2} \delta\phi_2 \tag{52}$$

For  $s = -1$ , the perturbation equation (2.5) of [22] is

$$\begin{aligned}
&\frac{1}{r^2} \partial_t^2 \Psi^{(-1)} - \frac{2}{r} \partial_t \partial_\phi \Psi^{(-1)} + \left(1 - \frac{(1 + \cos^2 \theta)^2}{4 \sin^2 \theta}\right) \partial_\phi^2 \Psi^{(-1)} \\
&- r^2 \partial_r^2 \Psi^{(-1)} - \frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta \Psi^{(-1)}) + \frac{2}{r} \partial_t \Psi^{(-1)} \\
&+ 2\left(i \frac{\cos \theta}{\sin^2 \theta} + i \frac{\cos \theta}{2}\right) \partial_\phi \Psi^{(-1)} + (\cot^2 \theta + 1) \Psi^{(-1)} = -\mathcal{J}_2
\end{aligned} \tag{53}$$

where  $\mathcal{J}_2 = 2\pi 2M^2 \eta^{-2} J_2 / (\eta \bar{\eta})$ . According to the symmetry of the background geometry, we use the following expansion

$$\Psi^{(-1)} = \sum_{\ell, m} e^{im(\phi+3r_0 t/4)} S_\ell(\theta) R_{\ell m}(r), \tag{54}$$

$$2\pi \mathcal{T} = \eta \bar{\eta} \sum_{\ell, m} e^{im(\phi+3r_0 t/4)} S_\ell(\theta) T_{\ell m}(r), \tag{55}$$

where  $\mathcal{T} = 2M^2 \eta^{-2} J_2$  and  $S_\ell$  are the spin-weighted spheroidal harmonics obeying

$$\frac{1}{\sin \theta} \partial_\theta (\sin \theta \partial_\theta S_\ell) + \left( K_\ell - \frac{m^2 + 1 - 2m \cos \theta}{\sin^2 \theta} - \frac{m^2}{4} \sin^2 \theta + m \cos \theta \right) S_\ell = 0, \tag{56}$$

The radial equation is

$$r^2 \partial_r^2 R_{\ell m} + (2m^2 - K_\ell + \frac{2\omega(m+i)}{r} + \frac{\omega^2}{r^2}) R_{\ell m} = T_{\ell m} \tag{57}$$

where

$$T_{\ell m} = \frac{ir_0^2 q}{4\sqrt{2}M^3} \left[ (2iS'_\ell(\frac{\pi}{2}) - \frac{3}{4}imS_\ell(\frac{\pi}{2}))\delta(r-r_0) + r_0S_\ell(\frac{\pi}{2})\delta'(r-r_0) \right] \quad (58)$$

From the result of gravitational perturbation [2], the solution of radial equation (57) satisfying the physical boundary condition is

$$R_{\ell m}(r) = \frac{1}{r_0^2 W} [\mathcal{X} \Theta(r_0 - r) \mathcal{W}(r) + \mathcal{Z} \Theta(r - r_0) \mathcal{M}(r)], \quad (59)$$

where  $W = 2i\omega \frac{\Gamma(2h)}{\Gamma(h-im+1)}$  and

$$\begin{aligned} \mathcal{X} &= r_0 \mathcal{M}'(r_0)(-a_1) + \mathcal{M}(r_0)(a_0 + 2a_1) \\ \mathcal{Z} &= \mathcal{X}(\mathcal{M} \rightarrow \mathcal{W}) \end{aligned} \quad (60)$$

And  $a_0$  and  $a_1$  is followed from (58), i.e.

$$\begin{aligned} a_0 &= \frac{ir_0^2 q}{4\sqrt{2}M^3} \left( -\frac{3imS}{4} + 2iS' \right) \\ a_1 &= \frac{ir_0^2 q}{4\sqrt{2}M^3} S \end{aligned} \quad (61)$$

The asymptotic behavior of the solution is immediately from [2]

$$\Psi^{(-1)}(r \rightarrow 0) = \sum_{\ell, m} e^{im(\phi+3r_0 t/4)} S_\ell(\theta) \frac{\mathcal{X}}{r_0^2 W} (-2i\omega)^{im-1} r^{-im+2} e^{-3imr_0/4r}, \quad (62)$$

$$\Psi^{(-1)}(r \rightarrow \infty) = \sum_{\ell, m} e^{im(\phi+3r_0 t/4)} S_\ell(\theta) \frac{\mathcal{Z}}{r_0^2 W} (-2i\omega)^h r^{-h+1}. \quad (63)$$

where  $h \equiv \frac{1}{2} + \sqrt{1/4 + K_\ell - 2m^2}$  and we assume  $h$  is real. The photon number flux at the horizon is given by [20]

$$\begin{aligned} \mathcal{F}_{\ell m} &= \frac{\frac{3}{4}r_0 M^6}{|\mathcal{B}|^2} m e^{-\pi m} \left| \frac{\mathcal{X}}{r_0^2 W} \right|^2 \\ |\mathcal{B}|^2 &\equiv (K_\ell - m^2)^2 + m^2 \end{aligned} \quad (64)$$

## 4.1 CFT analysis

The source term in the dual CFT can be read from the Herz potential of electromagnetic perturbation, which is given by

$$\Psi_H = \frac{1}{\mathcal{B}} R^{(-1)}(r) S^{(+1)}(\theta) e^{-i\omega t + im\phi} \quad (65)$$

where  $\omega = -\frac{3}{4}mr_0$ . And

$$\begin{aligned} R^{(-1)}(r) &= \frac{\mathcal{X}}{r_0^2 W} \mathcal{W}(r) \\ &\rightarrow \frac{\mathcal{X}}{r_0^2 W} \left[ \frac{(-2i\omega)^{1-h} \Gamma(2h-1)}{\Gamma(h+1-im)} r^h + \frac{(-2i\omega)^h \Gamma(1-2h)}{\Gamma(2-h-im)} r^{-h+1} \right] \quad \text{for } r \rightarrow \infty. \end{aligned} \quad (66)$$

So the source term is

$$J_{\ell m} = \frac{1}{\mathcal{B}} \frac{\mathcal{X}}{r_0^2 W} \frac{(-2i\omega)^{1-h} \Gamma(2h-1)}{\Gamma(h+1-im)} \quad (67)$$

Then the transition rate is

$$R_{\ell m} = 2\pi |J_{\ell m}|^2 C_{\mathcal{O}}^2 \frac{1}{\Gamma(2h_L)} e^{-\pi m} |\Gamma(h_L + im)|^2 \frac{2\pi}{\Gamma(2h_R)} \left(\frac{3}{4}mr_0\right)^{2h_R-1} \quad (68)$$

According to Kerr/CFT [9], we have

$$h_R = h, \quad h_L = h + 1. \quad (69)$$

If we choose the normalization factor

$$C_{\mathcal{O}}^2 = \frac{2^{2(h-1)} M^6}{(2\pi)^2} \frac{\Gamma(2h+2)\Gamma(2h)}{\Gamma(2h-1)^2}, \quad (70)$$

we can get the bulk result (64).

## 5 Conclusion

In this short note, we calculate the particle flux number of a massless neutral scalar produced by a orbiting star in the NHEKN geometry from the bulk gravity and the boundary CFT and find the agreement between them. We also investigate the electromagnetic radiation along with [2]. This is a little trivial case, it would be interesting to consider the electrical charged scalar waves or gravitational waves in the NHEKN geometry, and compare with calculations of the CFT.

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